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Hamiltonian Cycles in 4-Connected 4-Regular Claw-free Graphs

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Resumo

Since the decision problem of the hamiltonian cycle problem is NP-Complete, one recent trend has been to search for long cycles or related structures. In this aspect, a hamiltonian prism is an interesting relaxation of a hamiltonian cycle [2]. The *prism over a graph* G is the Cartesian product $G\Box K_2$ of G with the complete graph on two vertices. A prism can be seen as the graph obtained by joining the corresponding vertices of two copies of G. A graph G is *prism-hamiltonian* if its prism has a hamiltonian cycle.

Plummer [3] has conjectured that every 4-connected 4-regular claw-free graph is hamiltonian and this conjecture remains open [1]. Also, the author has shown that 4-connected 4-regular claw-free graphs fall into three classes $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$, of which only $\mathcal{G}_1$ is known to be hamiltonian. In our work, we prove that $\mathcal{G}_0$ is hamiltonian and that $\mathcal{G}_2$ is prism-hamiltonian, also corroborating to a conjecture that the prism over every 4-connected 4-regular graph is hamiltonian [2].

Given a graph G, let $G^1 = G\Box K_2$ and $G^q = G^{q-1}\Box K_2$, for $q > 1$. We show that, for every connected graph G, it holds that G^q is hamiltonian for all $q \geq [log_2 \Delta(G)]$, where $\Delta(G)$ is the maximum degree of G. Also, we show that this proof is equivalent to prove that $G\Box Q_n$ is prism-hamiltonian for some value of n where Q_n is the n-cube graph.

Referências

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[3] M. D. Plummer. *A note on Hamilton cycles in claw-free graphs*. Congressus Numerantium, 96113-122, 1993.